

### 2.20.1. Semantic Problems: Validity, Consistency, and Counterexample Sets

1. It was noted in 2.19 that any sentence follows validly from itself. Use **counterexample sets** and **(in)consistency** to explain why this is so.
2. It was also noted that, beginning with a valid argument, adding further premises always yields a (bigger) valid argument. For example, since the following familiar argument is valid,

$$\begin{array}{l} 1. (P \vee Q) \\ 2. \sim P \\ \hline \therefore Q \end{array}$$

this next argument is valid as well.

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$$\begin{array}{l} 1. (P \vee Q) \\ 2. \sim P \\ 3. X \\ \hline \therefore Q \end{array}$$

Use **counterexample sets** and **(in)consistency** to explain why adding further premises always leaves the (larger) argument valid.

*(Hint: use the features of consistent and inconsistent sets discussed in 2.17)*

3. It was also noted that an argument whose **conclusion** is a **tautology** is sure to be valid. (In other words: a tautology follows validly from any premise(s).) Use **counterexample sets** and **(in)consistency** to explain why this is so.
4. It was also noted that an argument with inconsistent premises is sure to be valid. Use **counterexample sets** and **(in)consistency** to explain why this is so.

5. Show that if an argument is invalid, then removing one or more premises will always yield a (smaller) argument which is also invalid.

*(Hint: use the features of consistent and inconsistent sets discussed in 2.17)*

6. Show that **adding a tautology** as an additional premise will **not affect** the **validity** of the argument – that is, if the original argument is invalid, adding a tautology as an additional premise will yield a (new, bigger) argument that’s also invalid; and if the original argument is valid, adding a tautology as an additional premise will yield a (new, bigger) argument that’s also valid.

*(Hint: first show that adding a tautology won’t affect the (in)consistency of a set. See also 2.18.1 Problem A15)*

7. The **(in)validity of an argument is not affected by adding an additional premise that is entailed by the existing premise(s)**. So, for example, since the left argument is invalid, the right one is too – because the added premise, “ $\sim P$ ,” is entailed by the original premise “ $\sim(P \vee Q)$ ”.

INVALID	INVALID
$  \begin{array}{l}  1. \sim (P \vee Q) \\  \hline  \therefore Q  \end{array}  $	$  \begin{array}{l}  1. \sim (P \vee Q) \\  2. \sim P \\  \hline  \therefore Q  \end{array}  $

Use this principle (along with the results from Problem (3) above) to explain the result reported in Problem (6) above.

8. (a) 2.18.1 Problem C showed that a consistent sentence never entails a contradiction. (Equivalently: that **the only (sort of) sentence that entails a contradiction is a contradiction.**)

That means the following principle holds, for one-premise arguments.

**(1) A contradiction follows validly only from a contradiction.**

Explain how Principle (1) can be strengthened to hold for arguments with any number of premises (so long as those sentences form an inconsistent set) – yielding Principle (2).

**(2) A contradiction follows validly only from an inconsistent set of sentences.** (Equivalently: **no contradiction follows from a consistent set of sentences.**)

(b) Use the features of inconsistent premises and validity discussed in 2.20 to strengthen (2) into Principle (3).

**(3) A contradiction follows validly from every inconsistent set of premises, and only from an inconsistent set of premises.**

9. Note that for each of the following **inconsistent** sets of sentences, the **negation of any one sentence** in the set **follows validly** from the remaining sentences.

Example 1:  $\{(P \vee Q), \sim P, \sim Q\}$

All the following arguments are valid:

$(P \vee Q), \sim P \therefore \sim\sim Q$

$(P \vee Q), \sim Q \therefore \sim\sim P$

$\sim P, \sim Q \therefore \sim(P \vee Q)$

Example 2:  $\{P, \sim P, Q\}$

All the following arguments are valid:

$$P, \sim P \therefore Q$$

$$P, Q \therefore \sim \sim P$$

$$Q, \sim P \therefore \sim P$$

Example 3:  $\{(P \wedge \sim P), X\}$

All the following arguments are valid:

$$(P \wedge \sim P) \therefore \sim X$$

$$X \therefore \sim(P \wedge \sim P)$$

Appealing only to the defining features of **validity**, **validity counterexample**, and **inconsistency**, along with the semantics for **negations**, show that **any inconsistent set** can be translated into a valid argument in this way.

**10.** In 2.19.1 we noted that the following general form, though typically yielding invalid arguments when formal sentences fill the blanks, will in certain mutant, limit cases yield a valid argument.

$$1. ( \bullet \vee \blacktriangle )$$

$$2. \sim \bullet$$

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$$\therefore \star$$

Given the link noted in 2.20 between validity and inconsistency, make a similar argument that the following general logical form, while typically yielding consistent sentences when its blanks are filled by sentences, will yield an **inconsistent** sentence in certain unusual cases.

$$( \bullet \wedge \blacktriangle )$$